# Production Line Performance by Using Queuing Model 

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#### Abstract

The efficiency of a production system is mainly characterized by batch size and throughput. To ensure system efficiencies, batch size and throughput must be in conformity to achieve optimum utilization. This paper discusses the use of Queuing Network Theory to study the effect of batch size and throughput in optimizing resource utilization, particularly machine resources in a manufacturing system. A factory in the manufacturing industry performing assembly operations in its production lines was the focus of this study. The result of the study was that when batch size and throughput are increased, the utilization also increases proportionately. Bottleneck will occur when the capacity is not enough to meet the demand requirement.


Keywords: Assembling process, production line, queuing theory, utilization.

## 1. INTRODUCTION

The manufacturing industry has to strive continually in order to increase efficiency in production process so as to stay competitive and sustainable. Production process is executed at a certain production line which is commonly divided into three types. They are single model line, batch model line and mixed model line (Groover 1987). Each machine in the production line operates at particular cycle time. The efficiencies of a production operation in manufacturing system can be measured based on the utilization of production resource such as machines in a particular cycle time.
For certain operations, resources are not utilized at the optimum level and this will cause the cycle time to produce a product will be longer and also the throughput is not at the maximum as it supposed to be. Hence, Queuing Network theory can be applied to determine the ideal batch size and throughput for a particular production in order to optimize resource utilization.

Queuing Network is one of the analytical models to evaluate manufacturing system. Generally, there are two approaches to model manufacturing system, which are analytical models and simulation models. Analytical models can be categorized into a spreadsheet model and a queuing network model (Marsudi et al., 2009).

There are many studies by previous researchers that related to resource utilization in a manufacturing system, but most of these studies are not focussed directly on batch size and throughput parameters. Based on that fact, the objective of this study is to evaluate the effect of batch size and throughput in relation to optimize the resource utilization by using Queuing Network theory. This study used Proton Door

Sash LH/RH production data that obtained from Z Company in Malaysia to apply the Queuing Network theory.

## 2. RELATED WORKS

Some previous studies to improve the resource utilization of manufacturing system was conducted by Taylor et al. (1994), Hopp and Spearman (2004), Seraj (2008),Walid (2006), Gamberi et al. (2008), and Hajji, et al. (2011).

Great effort was taken to study a batch size and scheduling model for production lines such as that by Marchet et al. (2011) who presented an innovation model that can be used in initial phase of "pick-and-sort" OPS design.

Hussain and Drake (2011) said that previous similar studies have used control theoretic techniques and it has been pointed out that control theorists are unable to solve the batch size problem. Therefore, they applied system dynamic simulation to investigate the impact of various batch sizes on bullwhip effect.

Stadtler and Sahling (2013) presented a new model formulation for batch size and scheduling of multi-stage flow lines which works without a fixed lead-time offset and still guarantees a feasible material flow.

Gamberia et al. (2008) presented a new approach to evaluating the suitability of implementing a batch-production-oriented manufacturing line.

Hong et al. (2012) proposed an integrated batching and sequencing procedure called the indexed batching model (IBM), with the objective of minimizing the total retrieval time (the sum of travel time, pick time and congestion delays).

Mengfei Yu, et al. (2013) proposed an approximation model based on queuing network theory to analyze the impact of order batching and picking area zoning on the mean order throughput time in a pick-and-pass order picking system.

Pazoura and Meller (2013)analyzed the impact batch retrieval processing has on throughput performance for horizontal carousel systems that use automated storage and retrieval machines as robotic pickers.

Taylor et al (1994) used a capacity analysis model to determine the maximum product quantity at electronic assembling facilities. The analysis is conducted on a set of product that consists of existing products mixed with the detail design of new product. In the case where maximum production quantity is not enough, the design of the new product should be changed in order to avoid production process at critical or bottleneck resources utilization.

Gamberi et al. (2008) presented the evaluation of the implementation of a manufacturing line by comparing different layouts. His studied was focused on analytical model for multi-stage multiproduct production line without buffer. In particular, the proposed approach involves both a preliminary choice considering the production capacity utilization rate.

Hajji, et al. (2011)described the analytical approach with an experimental approach based on simulation modeling, design of experiment and response surface methodology, to control manufacturing systems including to control utilization parameter.

Walid (2006) addressed the issue of capacity estimation and improvement in a multi-product unreliable production line with finite buffers. The procedure allowed for the enumeration of the defined states that a station may have while processing the mix of products. Durations of service interruptions or downtimes were taken into account as the mean time to repair the failed. This approach complements a linear programming model by altering the production sequence and inserting fictive product at appropriate positions in the sequence. The modified model provided the expected cycle time of the unreliable production line.

Chincholkar et al. (2004) presented the analytical model for estimating the total manufacturing cycle time and throughput of the manufacturing system. The development of their model followed the standard decomposition approach for queuing network approximations (Buzacott and Shanthikumar, 1993). The model considered the case where parts examined at a downstream inspection station and then were used to determine when the upstream process is out of control. The manufacturing cycle time from the out of control process to the downstream inspection process influences the detection time that elapses until the out of control process is noticed and repaired. Because an out of control process produces more bad parts, the detection time affects the number of good parts produced and the throughput of the manufacturing system.

Herrmann et al. (2000) presented a manufacturing system model based on queuing network approximations for estimating the manufacturing cycle time and throughput of such systems. In particular, the model can be used to evaluate the placement of inspection stations in a process flow. This analytical model can provide insights into how the manufacturing system parameters (including processing times, arrival rate, and placement of an inspection station) affect manufacturing system performance (including total manufacturing cycle time and throughput). The important result of their study was that the increasing manufacturing cycle time at one workstation can reduce both total manufacturing cycle time and throughput.

Johnson (2003) concluded that the utilization of a workstation in a production line can be increased by reducing production batch size. Anyway, he did not discuss both batch size and throughput parameters effects on the utilization parameter.

Other previous studies and literatures have discussed the relationship between throughput, cycle time and other performance parameters in the production line of a manufacturing system (Dessouky et al. 2002, Merchant 1993, Karimah 2005, Solberg 1981, Gershwin 1994, Montgomery 2001, Krajewski 2005, Colledani and Tolio2009, Abdulziz et al. 2012).

Previous studies that related to resource utilization have been discussed above. Anyway as mentioned before, only a few of these studies have discussed directly on the effect of batch size and throughput parameters to optimize resource utilization.

The following symbols are used in this paper:
$I=$ the set of all products
$T_{i}=$ desired throughput of product i (parts per hour)
$B_{i}=$ job size of product $i$ at release
$C_{i}^{r}=\mathrm{SCV}$ of job interarrival times for product $i$
$J=$ the set of all stations
$n_{j}=$ the number of resources at station $j$
$m_{j}^{f}=$ mean time to failure for a resource at station j
$m \stackrel{r}{j}=$ mean time to repair for a resource at station $j$
$R_{i}=$ the sequence of stations that product $i$ must visit
$R_{i j}=$ the subsequence that precedes station $j$
$t_{i j}=$ mean part process time of product $i$ at station $j$
$c_{i j}^{t}=\mathrm{SCV}$ of the part process time
$s_{i j}=$ mean job setup time of product $i$ at station $j$
$c_{i j}^{S}=\mathrm{SCV}$ of the setup time
$y_{i j}=$ yield of product $i$ at station $j$
$y_{i j}^{s}=$ scrap yield of product $i$ at station $j$

The model first calculates, for each product, the processing time of each job at each station. It also calculates, for each station, the average processing time, weighted by each product's arrival rate. Finally, it modifies the aggregate processing times by adjusting for the resource availability.
$Y_{i j}=$ cumulative yield of product $i$ through $R_{i j}$
$Y_{i}=$ cumulative yield of product $i$ through $R_{i}$
$x_{i}=$ release rate of product $i$ (jobs per hour)
$A_{j}=$ availability of a resource at station $j$
$V_{j}=$ the set of products that visit station $j$
$t_{i j}^{+}=$total process time of product i at station j
$c_{i j}^{+}=\mathrm{SCV}$ of the total process time
$t_{j}^{+}=$aggregate process time at station j
$c_{j}^{+}=\mathrm{SCV}$ of the aggregate process time
$t_{j}^{*}=$ modified aggregate process time at station j
$c_{j}^{*}=\mathrm{SCV}$ of the modified aggregate process time

$$
\begin{align*}
& Y_{i j}=\prod_{k \in R_{i j}} y_{i j}  \tag{1}\\
& Y_{i}=\prod_{k \in R_{i}} y_{i j}  \tag{2}\\
& x_{i}=T_{i} /\left(B_{i} Y_{i}\right)  \tag{3}\\
& A_{j}=\frac{m_{j}^{f}}{m_{j}^{f}+m_{j}^{r}}  \tag{4}\\
& V_{j}=\left\{i \in I: j \in R_{i}\right\}  \tag{5}\\
& t_{i j}^{+}=B_{i}\left(Y_{i j}\right)\left(t_{i j}\right)+s_{i j}+\left(1-y_{i j}^{s}\right)\left(s_{i j}\right)  \tag{6}\\
& \left(t_{i j}^{+}\right)^{2} c_{i j}^{+}=B_{i} Y_{i j} t_{i j}^{2} c_{i j}^{t}+\left(1-y_{i j}^{s}\right) s_{i j}^{2} c_{i j}^{s} \tag{7}
\end{align*}
$$

$$
\begin{align*}
& t_{j}^{+}=\frac{\sum_{i \in V_{j}} x_{i} t_{i j}^{+}}{\sum_{i \in V_{j}} x_{i}}  \tag{8}\\
& \left(t_{j}^{+}\right)^{2}\left(c_{j}^{+}+1\right)=\frac{\sum_{i \in V_{j}} x_{i}\left(t_{i j}^{+}\right)^{2}\left(c_{i j}^{+}+1\right)}{\sum_{i \in V_{j}} x_{i}}  \tag{9}\\
& t_{j}^{*}=\frac{t_{j}^{+}}{A_{j}}  \tag{10}\\
& c_{j}^{*}=c_{j}^{+}+2 A_{j}\left(1-A_{j}\right) \frac{m_{j}^{r}}{t_{j}^{+}} \tag{11}
\end{align*}
$$

The arrival process at each station depends upon the products that visit the station. Some products are released directly to the station, while others arrive from other stations. The departure process depends upon the arrival process and the service process.
$V_{0 j}=$ the set of products that visit station $j$ first
$V_{h j}=$ the set of products that visit station $h$ immediately before $j$
$\lambda_{j}=$ total job arrival rate at station $j$
$\lambda_{h j}=$ arrival rate at station $j$ of jobs from station $h$
$q_{h j}=$ proportion of jobs from station $h$ that next visit station $j$
$c_{j}^{a}=\mathrm{SCV}$ of interarrival times at station $j$
$c_{j}^{d}=\mathrm{SCV}$ of interdeparture times at station $j$
$\lambda_{j}=\sum_{i \in V_{j}} x_{i}$
$\lambda_{h j}=\sum_{i \in V_{h j}} x_{i}$
$q_{h j}=\lambda_{h j} / \lambda_{h}$
$c_{j}^{d}=1+\frac{u_{j}^{2}}{\sqrt{n j}}\left(c_{j}^{*}-1\right)+\left(1-u_{j}^{2}\right)\left(c_{j}^{a}-1\right)$
$c_{j}^{a}=\sum_{h \in J}\left(\left(c_{h}^{d}-1\right) q_{h j}+1\right) \frac{\lambda_{h j}}{\lambda_{j}}+\sum c_{i}^{r} \frac{x_{i}}{\lambda_{j}}$

Solving the above set of equations yields the complete of $c_{j}^{a}$ and $c_{j}^{d}$ for all stations. If the shop is a flow shop, and all products visit the same sequence of stations, then we can renumber the stations $1,2, \ldots, J . V_{j}=I$ and $V_{j-1, j}=I$ for all stations, and the last equation can be simplified as follows:
$c_{1}^{a}=\frac{\sum_{i \in I} c_{i}^{r} x_{i}}{\sum_{i \in I} x_{i}}$
$c_{j}^{a}=c_{j-1}^{d}, 2 \leq j \leq J$
The approximation calculates the utilization and the average cycle time $\left(W_{s}\right)$ at each workstation. The utilization incorporates the capacity requirements, while the average cycle time reflects the congestion and the existing variability.
$u_{j}=$ the average resource utilization at station $j$
$C T_{j}^{*}=$ the average cycle time at station $j$
$u_{j}=\frac{\lambda_{j} t_{j}^{*}}{n_{j}}$
$C T_{j}^{*}=\frac{c_{j}^{a}+c_{j}^{*}}{2} q\left(n_{j}, u_{j}\right) t_{j}^{*}+t_{j}^{*}$
$q\left(n_{j}, u_{j}\right)$ is a coefficient that is derived from an exact analytical model for the $\mathrm{M} / \mathrm{M} / \mathrm{n}_{\mathrm{j}}$ queuing system. For instance, $q\left(1, u_{j}\right)=u_{j} /\left(1-u_{j}\right)$ and $q\left(2, u_{j}\right)=u_{j}^{2} /\left(1-u_{j}^{2}\right)$.

## 3. RESULTS AND DISCUSSION

The results are obtained from three different batch and throughput values which were manipulated by using Excel to obtain the most optimum resource utilization. That three different batch and throughput values are represented as Analysis 1, Analysis 2 and Analysis 3.

Analysis 1: Batch size $=50$, Throughput $=20$
Analysis 2: Batch size $=60$, Throughput $=25$
Analysis 3: Batch size $=80$, Throughput $=31$
These three type of analyses data are different from the initial data which are: Batch size $=40$, throughput $=18$. The utilization for three production lines (in this case production process is started from line E followed by line D2 and finally line Assembly) based on the initial data can be referred to Fig. 1, 2, and Fig. 3.

Based on the chart plotted, the machines in the lines E and D2 do not achieve the maximum or optimum resource utilization. The imbalance exists in the resources utilization of the three production lines (line E. Line D2, and line Assembly).


Fig. 1. Machines utilization in line E based on the initial data


Fig. 2. Machines utilization in line D2 based on the initial data


Fig. 3. Machines utilization in line Assembly line based on the initial data

The average utilization percentage of machines in lines E and D2 are low (less than $35 \%$ ) if comparing to the machines in line Assembly which have utilization nearly $58 \%$ at CO2 welding workstation. This CO2 welding workstation is called "Bottleneck".
"Bottleneck" can be defined as available capacity of a resource that limits an organization to produce a particular quantity of a product or to meet the changing of market demand. To abolish the 'bottleneck' in the Assembly line, the batch size and throughput should be adjusted to obtain the utilization below $100 \%$ for all machines. To study the relationship between batch size and throughput in optimizing the resource utilization, the values of batch size and throughput are varied for at least three different values for each parameter. Hence, for the next step, Analysis 1 until Analysis 3 is done by changing the batch size and throughput values. The result of Analysis 1 until Analysis 3 can be
referred to Figure 4, 5 and Fig. 6 for line E, line D2, and line Assembly, respectively.


Fig. 4. Utilization improvement in line E


Fig.5. Utilization improvement in line D2


Fig. 6. Utilization improvement in line Assembly

For analysis 1, after the batch size and throughput are increased, the utilization for each station increases and the 'bottleneck' occurred in the Assembly line also increase. Each machine shows the utilization less than $100 \%$ ( $u j<100 \%$ ) and it shows that each machine involved in the manufacturing system has enough capacity to meet the desired production rate. However, this result still does not meet the most optimum utilization. Thus, the ideal batch size and throughput must be obtained to achieve the optimum resource utilization.

For the second analysis (Analysis 2), the batch size is added to 60 units and throughput is changed to 25 units. Comparing to the previous result, each machine shows incremental percentage of utilization. Even the utilization is increased, this result is still does not achieve the most optimum
utilization. Therefore, Analysis 3 needs to be done to obtain the ideal batch size and throughput.

For the third analysis (Analysis 3), again, the batch size and throughput are increased where the batch size is supposed to be 80 units and the throughput is 31 units. By referring to Fig. 6, CO2 welding station has stated the maximum utilization that is $0.99(99 \%)$ and therefore no other addition should be done to the batch size and throughput even though the percentage of utilization for all stations especially in the line E and line D 2 have utilization percentage not more than $55 \%$. Theoretically, utilization of each station cannot exceed or equal to $100 \%$ because if this happen, the existing capacity is not enough to meet the desired production rate.
By referring to the graph plotted as shown at Fig. 4 until Fig. 6 above, it is clear that when the batch size and throughput are raised, utilization of each station is also increased. Nevertheless, even the optimum utilization has been achieved, there is still imbalance of utilization occurred in the production system.

## 4. CONCLUSIONS

This paper presented the effect of batch size and throughput to optimize the resource utilization of a manufacturing system by using Queueing Network theory. In particular, the manufacturing system studied in this study is a multiple production line that produces a single product. The presented analytical model has the capability to show how batch size and throughput affect the performance of a manufacturing system.

This study also found that cycle time of each station can affect the resource utilization in a particular workstation, where the higher the cycle time of a workstation, the higher is the resource utilization and this sometimes can cause bottleneck when the capacity is not enough to meet the demand requirement. To solve this problem, the quantity of the workstation having high cycle time can be increased to reduce the cycle time of the process so as to ensure that the utilization level of each line in the manufacturing system is balanced. Based on this study, it can be concluded that the batch size is proportional to the throughput in terms to optimize the resource utilization of a manufacturing system.

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